*Journal of Sound and Vibration* (1997) **201**(5), 633–635

# LETTERS TO THE EDITOR

## COMMENTS ON "RESONANCE CONDITIONS AND DEFORMABLE BODY CO-ORDINATE SYSTEMS"

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(*Received* 24 *June* 1996)

A recent work by Shabana [1] addressed an important issue on selecting different sets of modes for problems of elastic beams that undergo large rigid-body displacements. Extensive works on flexible bodies and rotating beams have been conducted with different assumptions of boundary conditions for the beam's ends (sample lists can be found in references [1–3]). The classical models with clamped and pinned ends are commonly used in these works. In dealing with the related topic, Shabana [1] demonstrated that different mode shapes that correspond to different sets of natural frequencies can be used to obtain the same resonance conditions by using simple co-ordinate transformations. Two classical beam models with simply supported and free-free ends were first considered [1]. The relationship between the boundary conditions and the co-ordinate systems was discussed. The equations of beam vibration were then obtained by using the following two mode shapes: $\phi_{ss}(x) = \sin(\pi x/l)$ and $\phi_{mff}(x) = \phi_{ff} - \phi_{ff}(0)$, where $\phi_{mff} = \cos(\lambda x/l) + \cosh(\lambda x/l) - \sigma[\sin(\lambda x/l) + \sinh(\lambda x/l)]$ and $\phi_{ff}(0) = 2$. The modified mode shape $\phi_{ff}$ was generated from the classical free-free shape by defining a new co-ordinate system with a rigid translation, $\phi_{ff}(0)$. The two shape models were then applied to derive the uncoupled equation of motion in terms of modal co-ordinates: $m_j\ddot{q}_j + k_j q_j = Q_j$, where $m_j = \int_0^l \rho a \phi_j^2 \, \mathrm{d}x$, $k_j = \int_0^l EI_z(\partial^2\phi_j/\partial x^2)^2 \, \mathrm{d}x$, and $Q_j = \int_0^l F\phi_j \, \mathrm{d}x$ [4].

Assuming the beam is subjected to a harmonic force, $F_0 \sin \omega_f t$, acting at its center, the final equation of motion associated with the first mode of vibration was given by $\ddot{q} + \omega^2 q = B(F_0/m)\sin\omega_f t$, where $B$ is a parameter defined for later discussion. The results for the simply supported and modified free-free cases were given as [1]

$$\omega_{ss}^2 = (3{\cdot}142/l)^4\,(EI_z/\rho a), \quad B_{ss} = 2 \text{ and } \omega_{mff}^2 = (3{\cdot}163/l)^4(EI_z/\rho a), \quad B_{mff} = -0{\cdot}64312.$$

Furthermore, the solutions for $q$ were obtained [1]:

$$q_{ss} = \frac{B_{ss}F_0/(3{\cdot}142)^4(EI_z/l^3)}{1 - \omega_f^2/\omega_{ss}^2}\sin\omega_f t, \qquad q_{mff} = \frac{B_{mff}F_0/(3{\cdot}163)^4(EI_z/l^3)}{1 - \omega_f^2/\omega_{mff}^2}\sin\omega_f t.$$

The beam deflection at the midpoint was then evaluated by using $v|_{x=l/2} = \phi|_{x=l/2}q(t)$. It was mentioned that the physical displacements using the two different sets of modes are in good agreement [1]. In fact, the steady-state response of the simply supported beam can be obtained by the Duhamel integral [5, p. 438],

$$v = [2F_0\phi|_{x=l/2}/(m\omega^2)][\phi\sin\omega_f t/(1 - \omega_f^2/\omega^2)].$$

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0022–460X/97/150633 + 03 $25.00/0/sv960701 

Another parameter, $C|_{x=l/2} = \phi|_{x=l/2}B$ is now defined. In dealing with the same beam but having the force acting at other positions, the following results are found:

$$x/l = 1/10, \quad C_{ss} = 0{\cdot}191 \quad \text{and} \quad C_{mff} = 0{\cdot}171, \quad \Rightarrow \quad C_{mff}/C_{ss} = 0{\cdot}897;$$

$$x/l = 1/8, \quad C_{ss} = 0{\cdot}293 \quad \text{and} \quad C_{mff} = 0{\cdot}266, \quad \Rightarrow \quad C_{mff}/C_{ss} = 0{\cdot}907;$$

$$x/l = 1/6, \quad C_{ss} = 0{\cdot}5 \quad \text{and} \quad C_{mff} = 0{\cdot}463, \quad \Rightarrow \quad C_{mff}/C_{ss} = 0{\cdot}926;$$

$$x/l = 1/4, \quad C_{ss} = 1 \quad \text{and} \quad C_{mff} = 0{\cdot}967, \quad \Rightarrow \quad C_{mff}/C_{ss} = 0{\cdot}967;$$

whereas from reference [1],

$$x/l = 1/2, \quad C_{ss} = 2 \quad \text{and} \quad C_{mff} = 2{\cdot}068, \quad \Rightarrow \quad C_{mff}/C_{ss} = 1{\cdot}034.$$

It can be seen that $C_{mff} \approx C_{ss}$ for the center-loaded beams, but the ratio $C_{mff}/C_{ss}$ decreases as the force is acting away from the center.

Next turning to the responses obtained by using the two models: $v_{ss} = \phi_{ss}q_{ss}$ and $v_{mff} = \phi_{mff}q_{mff}$, the final form of their ratio is found to be

$$v_r = v_{mff}/v_{ss} = 0{\cdot}9737C_{mff}(1 - 0{\cdot}01026\bar{\omega}^2)/[C_{ss}(1 - 0{\cdot}00999\bar{\omega}^2)],$$

in which $\bar{\omega}^2 = \omega_f^2 \rho a l^4/(EI_z)$ is a non-dimensional parameter. The response ratio is plotted in Figure 1 as functions of $\bar{\omega}^2$ for different loading positions. Again, the ratio associated with the force acting at $x = l/2$ is almost one, but those with other loading positions can be less than 0·87. Moreover, the concept of a floating frame introduced in reference [1] was only demonstrated in terms of fundamental mode vibration, not based on multi-modes models.

It is well-known that a clamped-free model can lead to a good approximation if the rotation of the hub is treated as a degree of freedom [6–8]. One may use the base rotation or translation as a separate degree of freedom in many cases.

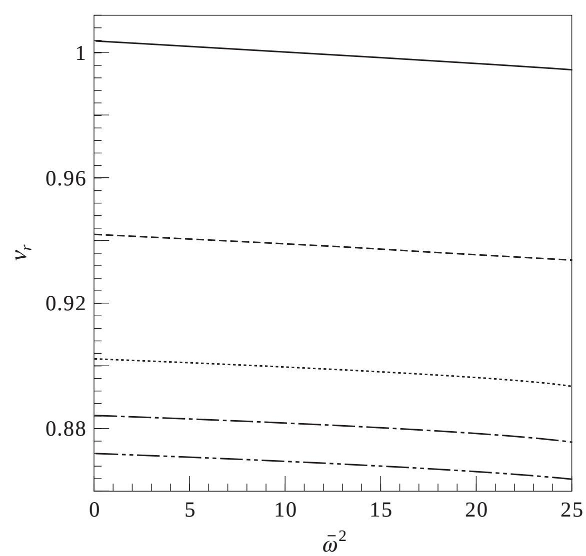


Figure 1. Response ratio versus frequency parameter for different loading positions: $x/l$: ——, 1/2; - - -, 1/4; · · · ·, 1/6; - · - · -, 1/8; - ·· - ·· -, 1/10.

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## AUTHOR'S REPLY

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(*Received* 22 *July* 1996)

The main objective of the work presented in reference [1] was to demonstrate that the natural frequency of the linear problem does not have a significant effect on the non-linear solution of the multibody dynamic equation. The analysis presented in this reference demonstrated the relationship between the end conditions and the selection of the deformable body co-ordinate system. It was demonstrated in reference [1] that different sets of modes that correspond to different sets of end conditions that define significantly different sets of natural frequencies can lead to *approximately* the same solution provided that similar deformation shapes are used. The results presented in Professor Low's letter confirm the conclusions obtained in reference [1] since these results can be used to demonstrate that the differences in the shapes can lead to some discrepancy in the results obtained using different sets of modes. The difference of 13% between the two models when the load applies close to the end of the beam is not proportional to the difference between the natural frequencies of the two models. Furthermore, close to the end of the beam, it is more likely that the deformation is small compared to the midpoint deflection, and therefore, the error of 13% may not be significant. It is also expected that more discrepancies can be found when the loads apply at points where the assumed shapes used in the two models have more significant differences.